

Gravitation as a Composite Particle Effect in a Unified Spinor-Isospinor Preon Field Model I

H. Stumpf

Institut für Theoretische Physik der Universität Tübingen

Z. Naturforsch. **43a**, 345 – 359 (1988); received February 5, 1988

The model is defined by a selfregularizing nonlinear preon field equation, and all observable (elementary and non-elementary) particles are assumed to be bound (quantum) states of fermionic preon fields. Electroweak gauge bosons, leptons, quarks, gluons as preon composites and their effective dynamics etc. were studied in preceding papers. In this paper gravitons are introduced as four-preon composites and their effective interactions are discussed. This discussion is performed by the application of functional quantum theory to the model under consideration and subsequent evaluation of a weak mapping procedure, both introduced in preceding papers. In the low energy limit it is demonstrated that the effective graviton dynamics lead to the complete homogeneous Einstein equations in tetrad formulation.

Key words: PACS 04.50 Unified field theories and other theories of gravitation, PACS 04.60 Quantum theory of gravitation, PACS 11.10 Field theory, PACS 12.10 Unified field theories and models, PACS 12.35 Composite models of particles

Introduction

The quantum theory of gravitation is one of the most challenging problems of modern physics. Due to the nonrenormalizability of the Hilbert-Einstein Lagrangian the suspicion grows stronger that the original Einstein equations are not the true microscopic description of gravitational phenomena. Thus in the last decades numerous approaches were made to replace the Hilbert-Einstein Lagrangian by a Lagrangian of a modified renormalizable field theory and to explain the classical theory of gravitation as a long wave or mean value approximation etc. Among the most prominent recent approaches are supergravity and superstrings which provide a modified theory of gravitation by means of gauging supersymmetric groups etc. [1, 2]. More conventional approaches consider gravity simply as an effect of a local non-abelian gauge group, such as the conformal group, the Poincaré group, the de Sitter group, etc. These approaches originate from the work of Weyl [3] and its extension by Yang and Mills [4] and led to numerous versions of nonrenormalizable or renormalizable gravitational theories. With respect to the Poincaré group, see for instance [5 – 10].

Reprint requests to Prof. H. Stumpf, Institut für Theoretische Physik der Universität Tübingen, Auf der Morgenstelle 14, D-7400 Tübingen.

General discussions of the gauge concept for gravity are contained in [11 – 16]. Theories with Lagrangians analogous to those of conventional gauge fields were first introduced by Weyl [3] and further developed along various lines, see for instance [17 – 32].

In spite of the enormous effort of applying modern group theory to derive modified gravitational theories, all the above-mentioned approaches can be considered as being conservative insofar as they do not doubt the role of the gravitational field as an elementary quantity. It is just this assumption which is abandoned in more radical approaches to explain gravity and quantum gravity. For instance, Bars and MacDowell [33] reproduced classical relativity as an approximation of a quantized field theory of interacting Rarita-Schwinger spinors and gauge potentials. Amati and Veneziano [34] proposed a rather complicated nonpolynomial nonlinear spinor field equation for Dirac-Spinors and approximately derived the Hilbert-Einstein action etc. Sen [35] used only spinors and spinor connections to describe classical relativity. D'Adda [36] derived the spinorfield Lagrangian of [34] by a symmetry breaking mechanism, and Saller [37] tried to give a deeper foundation of relativity theory by using Weyl-spinors with a Hermitean spinor metric, etc. Furthermore, maps between classical Einstein-equations and classical

0932-0784 / 88 / 0400-0345 \$ 01.30/0. – Please order a reprint rather than making your own copy.



Dieses Werk wurde im Jahr 2013 vom Verlag Zeitschrift für Naturforschung in Zusammenarbeit mit der Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V. digitalisiert und unter folgender Lizenz veröffentlicht: Creative Commons Namensnennung-Keine Bearbeitung 3.0 Deutschland Lizenz.

Zum 01.01.2015 ist eine Anpassung der Lizenzbedingungen (Entfall der Creative Commons Lizenzbedingung „Keine Bearbeitung“) beabsichtigt, um eine Nachnutzung auch im Rahmen zukünftiger wissenschaftlicher Nutzungsformen zu ermöglichen.

This work has been digitalized and published in 2013 by Verlag Zeitschrift für Naturforschung in cooperation with the Max Planck Society for the Advancement of Science under a Creative Commons Attribution-NoDerivs 3.0 Germany License.

On 01.01.2015 it is planned to change the License Conditions (the removal of the Creative Commons License condition “no derivative works”). This is to allow reuse in the area of future scientific usage.

Yang-Mills field equations, nonlinear σ -models etc. were discussed, for instance, by Sanchez [38] and Percacci [39].

The drawback of these unconventional approaches apparently is their ad hoc character. A more systematic approach is the concept of fusion of elementary fermions. In this concept all observable elementary particles are assumed to be bound states of the elementary fermions and the existence and the processes of observable particles are governed by the formation and reactions of these bound states. For relativistic particles this hypothesis was inaugurated by Jordan [40] who inferred the compositeness of the photon by a statistical argument. Subsequently, de Broglie [41] put forth the proposal that the photon is composed of a neutrino and an antineutrino. This was further developed by Jordan [42], Kronig [43] and other authors. Later on, de Broglie [44] extended his proposal to a general theory of fusion of particles of spin $1/2$, and along these lines Proca [45] and Kemmer [46] derived wave equations for composite mesons. In this context it was assumed that gravity is generated by a spin-2-field or particle, resp., the graviton, and Tonnelat [47], Petiau [48] and the Broglie [44] already discussed a composite graviton, i.e. a graviton composed of four spin- $1/2$ -fermions. A more formal approach to higher spin wave equations was performed by Dirac [49], Fierz [50], Pauli and Fierz [51], Bargmann and Wigner [52] and other authors. All these equations only describe local one-particle states, and in particular the graviton was “free”. With the further development of relativistic quantum field theory it became obvious that fusion need not be strictly local but can be a nonlocal phenomenon and that interaction has to be taken into account. However, the theory of the composite graviton did not participate in this general progress of quantum field theory, perhaps because this problem was too complicated or the research concentrated on other fields. The only attempts to include a composite graviton into current research were made by Bopp [53] who generalized de Broglie’s ansatz to many-particle equations with relativistic potentials and Heisenberg [54] who assumed any elementary particle to be a bound state of elementary spinor-isospinor fields in his nonlinear spinor field approach. But neither these two authors nor their co-workers were able to derive quantitative conclu-

sions about a composite graviton. Later on, Kurdgelaidze and Funke [55] and Danilyuk [56] made first quantitative calculations of composite spin-2-mesons and some elementary graphs in the framework of a nonlinear spinor theory, while Dürr [57] and Saller [58] made some speculations about the generation of a composite graviton by symmetry breaking effects. From these papers no stringent conclusions can be drawn, whether the composite graviton is meaningful or not and which dynamical laws govern its reactions.

Recently there has been a renewed interest in the group theoretical analysis of graviton states of the Bargmann-Wigner equations (de Broglie! equations) by Nous [59], Rodriguez and Lorente [60] and Doughty and Collins [61]. As these equations describe only free pointlike gravitons the question arises whether these “free particle” gravitons can be used as building blocks for the construction of the full gravitational Einstein equations or not. This question was answered affirmatively. Starting with free pointlike gravitons in Minkowski space it was shown by Gupta [62], Kraichnan [63], Arnowitt [64], Weinberg [65], Wyss [66], Ogievetsky and Polubarinov [67], Deser [68], Groenewold [69], Boulware and Deser [70], Papini and Valluri [71], von der Bij, van Dam and Ng [72], that interactions of such gravitons necessarily lead to the Einstein equations. Furthermore, it was demonstrated by Mittelstaedt [73] that even cosmological solutions can be obtained from this formulation of Einstein’s theory in Minkowski space. This means: the assumption of the existence of gravitons does not contradict general relativity. Therefore, the attempt to derive Einstein’s equations from a theory of composite gravitons is neither selfcontradictory. In the following we demonstrate this for a spinor-isospinor preon field model, where gravitons arise from four-preon states.

The use of composite gravitons is of considerable advantage compared with point-like gravitons. While according to the above-mentioned authors the pointlike gravitons strictly lead to Einstein’s equations and thus a nonrenormalizable theory, the internal structure of composite gravitons leads to formfactors in their mutual interactions which prevent an ultraviolet catastrophe. In this way Einstein’s equations are a low-energy limit which just corresponds to the ideas men-

tioned at the beginning and the composite gravitons correspond to the most economic way of a modification of the laws of classical relativity.

Within the context of a unified preon field model [74–79] the dynamical laws which govern the reactions and interactions of composite particles are effective field equations which are derived from the preon field equations by means of a weak mapping procedure in functional space [75, 76, 80]. The weak mapping procedure for generating functionals is a mathematically well-defined method which is the quantum field theoretical analogon to the resonating group method in nuclear physics [81–86] and other nonrelativistic multiparticle branches. Both in nuclear physics and in quantum field theory the critical point are the wavefunctions of the composite particles which are used as elements of the map. With respect to such wavefunctions two-preon-states corresponding to electroweak gauge bosons and scalar bosons were quantitatively calculated in the framework of the above mentioned preon model [74, 79]. The four preon states have at present not yet been quantitatively investigated. Thus we propose the internal structure of these states guided by the results of [79] and of investigations of the Bargmann-Wigner equations [59–61]. The papers of Lord [87], Wilson [88], Dehnen and Ghaboussi [89] which at first sight are only loosely connected to that problem provided hints for controlling and justifying our assumptions.

Finally, in particular in connection with general relativity, some authors, for instance Gürsey [90], Braunss [91], Datta [92], Hehl and Datta [93], Ulmer [94], Borneas [95], have derived spinor field equations from torsion or more general principles etc. We do not exclude the possibility of deriving spinor field equations from a more basic principle. But at present we follow the de Broglie-Bopp-Heisenberg program to derive the entire high-energy phenomenology by the fusion of selfinteracting elementary fermions in Minkowski space.

1. Fundamentals of the Model

The unified preon field model which is assumed to be the basis of the theory is defined by the second order derivative nonlinear field equation

$$\begin{aligned} & [(-i\gamma^\mu \partial_\mu + m_1)(-i\gamma^\nu \partial_\nu + m_2)]_{\alpha\beta} \psi_\beta(x) \\ & = g V_{\alpha\beta\gamma\delta} \psi_\beta(x) \bar{\psi}_\gamma(x) \psi_\delta(x), \end{aligned} \quad (1.1)$$

where the indices α, β, \dots are superindices describing spin and isospin. Due to the mass terms in (1.1) the corresponding spinor field has to be a Dirac-spinor-isospinor.

In contrast to the nonrenormalizability of first order derivative nonlinear spinor field equations and the difficulties connected with this property, the model (1.1) exhibits self-regularization, relativistic invariance and locality for common-canonical quantization. Due to the self-regularization the model is renormalizable, but we need not make use of this property on account of our nonperturbative calculation techniques. In particular, it was demonstrated by direct nonperturbative calculations [74, 79] that relevant matrix elements of local composite operators as occurring in (1.1) are finite and need no special renormalization apart from normal ordering.

The regularization of the spinor field dynamics by (1.1) leads to indefinite metric in the corresponding state space. It was, however, shown [74, 79] that for observable states the probability interpretation can be maintained. For the further evaluation equation (1.1) has to be decomposed into an equivalent set of first order derivative equations. It was proved by the author [96] and Grosser [97] that the set of nonlinear equations $r = 1, 2$

$$\begin{aligned} & (-i\gamma^\mu \partial_\mu + m_r)_{\alpha\beta} \varphi_{\beta r}(x) \\ & = g \lambda_r \sum_{stu} V_{\alpha\beta\gamma\delta} \varphi_{\beta s}(x) \bar{\varphi}_{\gamma t}(x) \varphi_{\delta u}(x) \end{aligned} \quad (1.2)$$

is connected with (1.1) by a biunique map where this map is defined by the compatible relations

$$\begin{aligned} \psi_\alpha(x) &= \varphi_{\alpha 1}(x) + \varphi_{\alpha 2}(x), \\ \varphi_{\alpha 1}(x) &= \lambda_1 (-i\gamma^\mu \partial_\mu + m_2)_{\alpha\beta} \psi_\beta(x), \\ \varphi_{\alpha 2}(x) &= \lambda_2 (-i\gamma^\mu \partial_\mu + m_1)_{\alpha\beta} \psi_\beta(x) \end{aligned} \quad (1.3)$$

with $\lambda_r := (-1)^r (2\Delta m)^{-1}$ and $\Delta m = \frac{1}{2}(m_1 - m_2)$.

According to [76] the vertex operator must have the form

$$V_{\alpha\beta\gamma\delta} = \frac{1}{2} \sum_{h=1}^2 (v_{\alpha\beta}^h v_{\gamma\delta}^h - v_{\alpha\delta}^h v_{\beta\gamma}^h) \quad (1.4)$$

with $\alpha \equiv (\alpha, A) \equiv (\text{spinor, isospinor index})$ etc., and

$$\begin{aligned} v_{\alpha\beta}^h &\equiv \hat{v}_{\alpha\beta}^h \delta_{AB}, \quad h = 1, 2, \\ \hat{v}_{\alpha\beta}^1 &:= \delta_{\alpha\beta}, \quad \hat{v}_{\alpha\beta}^2 := i\gamma_{\alpha\beta}^5. \end{aligned} \quad (1.5)$$

This kind of coupling is needed to obtain non-abelian effective field theories for composite particles.

It is furthermore convenient to replace the adjoint spinor by the charge conjugated spinor [76]. The charge conjugated spinor (isospinor) is defined by

$$\varphi_{Ajv}^c = C_{vv'}^{-1} \bar{\varphi}_{Ajv'} \quad (1.6)$$

and introducing the superspinors

$$\varphi_{Aja1} := \varphi_{Aja}, \quad \varphi_{Aja2} := \varphi_{Aja}^c, \quad (1.7)$$

we can combine (1.2) and its charge conjugated equation into one equation [76]

$$\begin{aligned} &\sum_{Z_2} (D_{Z_1 Z_2}^\mu \partial_\mu - m_{Z_1 Z_2}) \varphi_{Z_2} \\ &= \sum_{h Z_2 Z_3 Z_4} U_{Z_1 Z_2 Z_3 Z_4}^h \varphi_{Z_2} \varphi_{Z_3} \varphi_{Z_4} \end{aligned} \quad (1.8)$$

with $Z := (\alpha, A, i, A)$ and

$$\begin{aligned} \alpha &= \text{spinor index } (\alpha = 1, 2, 3, 4), \\ A &= \text{isospinor index } (A = 1, 2), \\ i &= \text{auxiliary field index } (i = 1, 2), \\ A &= \text{superspinor index } (A = 1, 2), \end{aligned} \quad (1.9)$$

where the following definitions are used

$$\begin{aligned} D_{Z_1 Z_2}^\mu &:= i\gamma_{\alpha_1 \alpha_2}^\mu \delta_{A_1 A_2} \delta_{i_1 i_2} \delta_{A_1 A_2}, \\ m_{Z_1 Z_2} &:= m_{i_1} \delta_{\alpha_1 \alpha_2} \delta_{A_1 A_2} \delta_{i_1 i_2} \delta_{A_1 A_2}, \\ U_{Z_1 Z_2 Z_3 Z_4}^h &:= g \lambda_{i_1} \hat{v}_{\alpha_1 \alpha_2}^h \delta_{A_1 A_2} \delta_{A_1 A_2} (\hat{v}^h C)_{\alpha_2 \alpha_3} \\ &\quad \cdot \delta_{A_3 A_4} \delta_{A_3 A_1} \delta_{A_4 A_2}. \end{aligned} \quad (1.10)$$

The quantum states of the model (1.1) or (1.8) respectively are described by state functionals $|\mathfrak{T}[j, a]\rangle$ with respect to the states $\{|a\rangle\}$ where $j \equiv j_Z(x)$ are sources with corresponding Z -indices. For concrete calculations it is necessary to introduce normal transforms by $|\mathfrak{T}\rangle = Z_0[j] |\mathfrak{F}\rangle$ and the energy representation of the spinor field in terms of state functionals. Both procedures were discussed in detail in [78] and yield a functional equation for $|\mathfrak{F}\rangle$. In this equation the limit to a

one-time description can be performed, so that eventually the following equation results

$$\begin{aligned} p_0 |\mathfrak{F}\rangle &= \sum_{I_1 I_2} j_{I_1} K_{I_1 I_2} \partial_{I_2} |\mathfrak{F}\rangle \\ &\quad + \sum_{h I_1 I_2 I_3 I_4} j_{I_1} W_{I_1 I_2 I_3 I_4}^h d_{I_4} d_{I_3} d_{I_2} |\mathfrak{F}\rangle \\ &= \mathcal{H} |\mathfrak{F}\rangle \end{aligned} \quad (1.11)$$

if the abbreviations

$$d_I = \partial_I - \sum_{I'} F_{II'} j_{I'} \quad (1.12)$$

and

$$\begin{aligned} K_{I_1 I_2} &:= K_{Z_1 Z_2}(\mathbf{r}_1, \mathbf{r}_2) \\ &:= i \sum_Z D_{Z_1 Z}^0 (D_{ZZ_2}^k \partial_k^1 - m_{ZZ_2}) \delta(\mathbf{r}_1 - \mathbf{r}_2) \end{aligned} \quad (1.13)$$

$$\begin{aligned} W_{I_1 I_2 I_3 I_4}^h &= W_{Z_1 Z_2 Z_3 Z_4}^h(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \\ &:= i \sum_Z D_{Z_1 Z}^0 U_{ZZ_2 Z_3 Z_4}^h \\ &\quad \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(\mathbf{r}_1 - \mathbf{r}_3) \delta(\mathbf{r}_1 - \mathbf{r}_4) \end{aligned} \quad (1.14)$$

are introduced. Summation over $I \equiv$ summation over Z , integration over r !

We assume the spinor field interaction term to be normalordered. Then the local terms $F_{I_2 I_3} \partial_{I_4}$ etc. drop out and we obtain a well-defined interaction from (1.11). The special form of this interaction term of (1.11) was given in [76].

2. Effective Graviton-Preon Dynamics

If gravitons are assumed to be composite preon states then the following requirements have to be fulfilled:

- i) In the low-energy limit the graviton selfcoupling has to coincide with classical relativity theory;
- ii) in the low-energy limit the graviton-matter coupling has to reproduce the classical couplings of the corresponding fields.

In the high energy limit deviations from the classical couplings are admitted and lead to formfactors in the interactions, see [75].

According to our model all kind of observable matter is build up of preons. For a first exploration of gravity as a composite particle effect we concentrate on the coupling of gravitons with these

elementary constituents. Because if their coupling satisfies ii), one can expect that due to the compositeness all other graviton-matter couplings will satisfy ii), too. We thus confine ourselves to graviton-preon systems.

We assume the graviton to be composed of four preons in agreement with the idea of spin fusion, see [44, 47, 48, 59–61]. In contrast to the solutions of the corresponding de Broglie-Bargmann-Wigner (\equiv BBW) equations our graviton states are, however, assumed to be nonlocal fusion states. The properties and the physical meaning of such composite preon states can be studied by the investigation of the effective interactions. In [75, 76] it was demonstrated that the appropriate method for such an investigation is the weak mapping procedure. This procedure is defined by a transformation of the set of functional preon source operators $\{j_I\}$ into a set of functional cluster source operators $\{X_R\}$. For the case of a graviton-preon system the corresponding cluster source operators for the bosons read

$$b_K := \sum_{I_1 I_2 I_3 I_4} C_K^{I_1 I_2 I_3 I_4} j_{I_1} j_{I_2} j_{I_3} j_{I_4}, \quad (2.1)$$

where the set of coefficient functions $\{C_K, K = 1 \dots\}$ is defined to be a complete set of four-preon cluster states, i.e. this set contains bound state clusters (gravitons) as well as scattering state clusters of four preons, etc. The fermion cluster operators are trivial $f_I \equiv j_I$ due to the restriction to preons.

The transformation of the preon source operators $\{j_I\}$ into boson and fermion source operators $\{b_K, f_I\}$ induces a transformation of the set of functional states $\{|\mathfrak{F}\rangle\}$ as well as of the corresponding functional equation (1.11). This transformation is characterized by the invariance conditions

$$|\mathfrak{F}[j, a]\rangle = |\mathfrak{F}[b, f, a]\rangle \quad (2.2)$$

and

$$\mathcal{H}\left[j, \frac{\delta}{\delta j}\right] = \hat{\mathcal{H}}\left[b, \frac{\delta}{\delta b}, f, \frac{\delta}{\delta f}\right], \quad (2.3)$$

where $|\mathfrak{F}\rangle$ and $\hat{\mathcal{H}}$ are the cluster transforms of $|\mathfrak{F}\rangle$ and \mathcal{H} of (1.11).

The weak mapping of a boson-fermion system with two-preon scalar boson states and three-preon fermion states was discussed in [75]. By a detailed

investigation it was demonstrated that the cluster state transform $\hat{\mathcal{H}}$ of the functional energy operator \mathcal{H} leads to a hierarchy of interactions which is generated by the different magnitudes of the coupling constants of the individual terms in \mathcal{H} . This property in combination with high preon masses allows the application of the leading term approximation. This approximation was thoroughly discussed in [75]. It is performed in two steps:

- i) The complete sets of cluster states, i.e. of scattering states as well as of bound states are reduced to the subsets of bound states;
- ii) the complete set of transformed functional energy operator terms is reduced to the subset of highest magnitude terms.

While the first step is justified by energetic estimates, the second step is justified by interaction estimates. Furthermore, it is convenient to assume that only the physically relevant bound states occur and that possibly exotic bound states can be excluded by more detailed bound state investigations. These assumptions were already partly verified by direct calculation [79] and are a peculiar feature of the model under consideration.

With respect to the weak mapping of the graviton-preon system we observe that the term hierarchy of \mathcal{H} is the same as that for the boson-fermion system. For a first exploration of the graviton-preon system we assume for brevity that the leading term approximation works also for this system in analogy to the boson-fermion system.

The cluster transform $\hat{\mathcal{H}}$ of \mathcal{H} allows the following decomposition

$$\begin{aligned} \hat{\mathcal{H}} = & \mathcal{H}_F \left[f, \frac{\delta}{\delta f} \right] + \mathcal{H}_B \left[b, \frac{\delta}{\delta b} \right] \\ & + \mathcal{H}_{BF} \left[f, \frac{\delta}{\delta f}, b, \frac{\delta}{\delta b} \right] \end{aligned} \quad (2.4)$$

where \mathcal{H}_F is the pure preon part, \mathcal{H}_B the pure graviton part, while \mathcal{H}_{BF} contains the interactions between both kinds of constituents. By direct calculation we obtain $\mathcal{H}_F \equiv \mathcal{H}$ and in the leading term approximation for the pure graviton part $\mathcal{H}_B = \mathcal{H}_B^0 + \mathcal{H}_B^I$ with

$$\begin{aligned} \mathcal{H}_B^0 := & 4 \sum_{\substack{I_1 I_2 I_3 I_4 I_1' \\ nm}} R_{I_1 I_2 I_3 I_4}^m K_{I_1 I_1'} C_n^{I_1' I_2 I_3 I_4} b_m \frac{\delta}{\delta b_n} \\ & - 6 \sum_{\substack{h I_1 I_2 I_3 I_4 \\ L_1 L_2 L_3 nm}} R_{I_1 I_2 I_3 I_4}^m W_{I_1 L_1 L_2 L_3}^h \\ & \cdot F_{L_3 I_2} C_n^{L_1 L_2 I_3 I_4} b_m \frac{\delta}{\delta b_n}, \end{aligned} \quad (2.5)$$

$$\begin{aligned} \mathcal{H}_B^I := & 24 \sum_{\substack{h L_1 L_2 L_3 nm n' \\ I_1 I_2 I_3 I_4 I_5 I_6}} R_{I_1 I_2 I_3 I_4 I_5 I_6}^m \hat{W}_{I_1 L_1 L_2 L_3}^h \\ & \cdot C_n^{L_3 I_4 I_5 I_6} C_{n'}^{L_1 L_2 I_2 I_3} b_m \frac{\delta}{\delta b_n} \frac{\delta}{\delta b_{n'}}, \end{aligned} \quad (2.6)$$

whereas the graviton-preon interaction is given by

$$\begin{aligned} \mathcal{H}_{BF} := & 6 \sum_{\substack{h L L_1 L_2 L_3 \\ I_2 I_3 k n}} C_n^{L_1 L_2 I_3 I_4} R_{L I_3 I_4}^k \hat{W}_{L L_1 L_2 L_3}^h \\ & \cdot f_k \frac{\delta}{\delta f_{L_3}} \frac{\delta}{\delta b_n}, \end{aligned} \quad (2.7)$$

where $R_{I_1 I_2 I_3 I_4}^m$ are the dual graviton states, while $R_{I_1 I_2 I_3 I_4 I_5 I_6}^m$ and $R_{I_1 I_2 I_3}^k$ are the first order polarization cloud states of gravitons and preons, resp. Furthermore, the definition

$$\hat{W}_{L L_1 L_2 L_3}^h = \{W_{L L_1 L_2 L_3}^h\}_{\text{as}(L_1 L_2 L_3)} \quad (2.8)$$

is used, i.e., the vertex is antisymmetrized in the last three superindices. In this paper we treat only the effective graviton selfcoupling which follows from evaluation of \mathcal{H}_B . In the next paper the graviton-preon coupling (2.7) will be discussed.

Due to the above mapping-procedure the functional energy equation (1.11) is mapped into the equation $E |\hat{\mathfrak{F}}\rangle = \hat{\mathcal{H}} |\hat{\mathfrak{F}}\rangle$ and if we confine ourselves to the graviton dynamics, the corresponding equation reads

$$\mathcal{H}_B |\hat{\mathfrak{F}}\rangle_B = (\mathcal{H}_B^0 + \mathcal{H}_B^I) |\hat{\mathfrak{F}}\rangle_B = E |\hat{\mathfrak{F}}\rangle_B. \quad (2.9)$$

This functional equation describes a nontrivial nonlinear quantum field theory which is rather difficult to evaluate. For a first exploration it is, however, sufficient to derive gravity from (2.9) as a mean value or as a classical approximation, resp.

This is achieved by means of an appropriate approximation of the state functional $|\hat{\mathfrak{F}}\rangle_B$. Since, apart from formfactors etc., (2.9) is to describe a conventional field theory of pointlike particles, we can apply a convenient approximation scheme which is provided by conventional quantum field theory for this case. Such a scheme is defined by the application of connected functional states. For gravitons the connected functional $Z_c[b]$ is defined by

$$|\hat{\mathfrak{F}}\rangle_B = e^{Z_c[b]} |0\rangle, \quad (2.10)$$

where $Z_c[b]$ is a power series in the graviton source operators b . The usual quantum field theoretic approximation techniques [98] are based on an a priori given Lagrangian of the field under consideration, a precondition which obviously is not fulfilled in our case, as the only a priori given Lagrangian is that of the preons and not that of gravitons. Thus we are forced to apply an approximation method of statistical mechanics to $Z_c[b]$ where the lowest approximation reads

$$Z_c[b] = \sum_n z_n b_n \quad (2.11)$$

with $\{z_n\}$ as the one-particle graviton transition matrix elements, i.e. wave functions. By means of this approximation in statistical mechanics classical hydrodynamic equations can be derived and we expect in our case to obtain the classical equations of gravity.

The operators (2.5) (2.6) can formally be written

$$\mathcal{H}_B^0 = \ell_B^k + \ell_B^p = \sum_{mn} [K_{mn} + P_{mn}] b_m d_n \quad (2.12)$$

and

$$\mathcal{H}_B^I = \sum_{mnn'} [V_{mnn'} b_m d_n d_{n'}]. \quad (2.13)$$

Substitution of (2.10) – (2.13) into (2.9) then gives the equation

$$\begin{aligned} e^{\sum z_l b_l} \left\{ \sum_{mn} [K_{mn} + P_{mn}] b_m z_n \right. \\ \left. + \sum_{mnn'} V_{mnn'} b_m z_n z_{n'} - \sum_m \frac{\partial}{\partial t} b_m z_m \right\} |0\rangle = 0. \end{aligned} \quad (2.14)$$

This equation is solved if the equation

$$b_m \left\{ \sum_{mn} [K_{mn} + P_{mn}] z_n + \sum_{mnn'} V_{mnn'} z_n z_{n'} - \sum_m \frac{\partial}{\partial t} z_m \right\} = 0 \quad (2.15)$$

is satisfied for arbitrary b_m , i.e., we obtain a set of nonlinear equations for the determination of z_n . In the following we will evaluate these equations.

3. Low Energy Graviton States

For the computation of the effective graviton dynamics by means of the weak mapping procedure the explicit form of the graviton wave functions is needed. As long as these wave functions are not derived from a direct calculation, they have to be postulated. Since the calculation of four preon bound states is rather complicated, in a first exploration we postulate them. For a physically meaningful guess of the graviton wave functions the assumption of being composed of four preons is too general. It must be supplied by a more detailed information. Such an information is provided by considering the pointlike composite graviton states which are solutions of the BBW-equations [44, 47, 48, 59–61], and by proposals contained in [87–89] which support this approach. For brevity we cannot draw the conclusions in detail. Summarizing the results we are forced to assume that graviton states are the direct product of two vector boson states. With this conclusion we can relate graviton states to detailed quantitative calculations. Vector boson states were extensively treated in [76, 79] as two-preon bound state solutions in the low energy approximation. In this approximation the dependence of the internal wave function on the center of mass momentum is neglected, and taking over such vector boson wave functions for the construction of graviton wave functions we work within the same approximation scheme.

According to [76, 79] the wave functions of the vector boson states are given by

$$C \left(\begin{array}{cc|c} r & r' & k \\ i & i' & \sigma \\ \alpha & \alpha' & t \\ \chi & \chi' & \end{array} \right) = U^{ii'} e^{ik(r+r')/2} \cdot \varphi(r-r') S_{\alpha\alpha'}^\sigma T_{\chi\chi'}^t \quad (3.1)$$

where U is the auxiliary field operator, S the symmetrical spinor basis and T the antisymmetrical super-spinor-isospinor basis. These quantities are discussed in detail in [76, 79]. The polarization amplitude of the center of mass-motion is not included in (3.1). In [76] it was rather shifted into the vector boson source operators themselves. In the following we will do the same for the graviton case.

By means of (3.1) a general bound state of two vector bosons can be written in the following form

$$C_n \left(\begin{array}{cccc} r_1 & r_2 & r_3 & r_4 \\ i_1 & i_2 & i_3 & i_4 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \chi_1 & \chi_2 & \chi_3 & \chi_4 \end{array} \right) = \int c(k, k') \left\{ C \left(\begin{array}{cc|c} r_1 & r_2 & k \\ i_1 & i_2 & \sigma \\ \alpha_1 & \alpha_2 & t \\ \chi_1 & \chi_2 & \end{array} \right) - \right. \\ \left. \cdot C \left(\begin{array}{cc|c} r_3 & r_4 & k' \\ i_3 & i_4 & \sigma' \\ \alpha_3 & \alpha_4 & t' \\ \chi_3 & \chi_4 & \end{array} \right) \right\} dk dk' \quad (3.2)$$

where the bracket means antisymmetrization in $I_1 I_2 I_3 I_4$. In order to have a definite center of mass momentum the wave function $c(k, k')$ must have the special form

$$c(k, k') = \delta(K - K') c(w) \quad (3.3)$$

with $k = \frac{1}{2} K' + w$, $k' = \frac{1}{2} K' - w$ and K the true center of mass momentum. Then (3.2) goes over into

$$C_n = e^{iK(r_1+r_2+r_3+r_4)/4} \{ \varphi^{i_1 i_2}(r_1 - r_2) \\ \cdot \varphi^{i_3 i_4}(r_3 - r_4) \hat{\varphi}(r_1 + r_2 - r_3 - r_4) \\ \cdot S^\sigma S^{\sigma'} T^t T^{t'} \}_{\text{as}} \quad (3.4)$$

where $\hat{\varphi}$ is the Fourier transform of $c(w)$.

If (3.4) is to describe a graviton state, in the pointlike approximation this function must reduce to the direct product of two pointlike vector boson states. We are going to show this by evaluation of the diagonal part of \mathcal{H}_B , which corresponds to the common BBW-equation.

For the evaluation of \mathcal{H}_B^0 the diagonal part (2.5) of \mathcal{H}_B , the dual states R^m are needed. These states can be obtained from (3.1) and (3.2) by replacing φ by its dual counterpart σ_3 , i.e. the left-hand solution of the φ -eigenvalue equation, see [74, 79]. The evaluation of \mathcal{H}_B^0 itself by substitution of C_n and R^m into (2.5) is straightforward. In the leading

term approximation the exchange integrals can be neglected. The techniques of evaluation were extensively discussed in [75, 76]. So here we will not repeat explicit calculations, but give only their results.

Due to (3.4) we observe that in full notation we can write $b_m \equiv b_{\lambda u, \lambda' u'}$ and $d_n \equiv d_{\sigma t, \sigma' t'}$ and define

$$\begin{aligned} b_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}^{uu'}(\mathbf{r}) &:= \sum_{\lambda \lambda'} [S_{\alpha_1 \alpha_2}^\lambda S_{\alpha_3 \alpha_4}^{\lambda'}]_{\text{sym}(\lambda, \lambda')} b_{\lambda u, \lambda' u'}(\mathbf{r}), \\ d_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}^{tt'}(\mathbf{r}) &:= \sum_{\sigma \sigma'} [S_{\alpha_1 \alpha_2}^\sigma S_{\alpha_3 \alpha_4}^{\sigma'}]_{\text{sym}(\sigma, \sigma')} d_{\sigma t, \sigma' t'}(\mathbf{r}). \end{aligned} \quad (3.5)$$

By strictly observing antisymmetry properties etc. and using definitions (3.5) we obtain for (2.5)

$$\begin{aligned} \mathcal{H}_B^0 &= \sum_{tt'} \int b_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}^{tt'}(\mathbf{r}) \{[(\alpha^k \partial_k - M\beta - g\beta)_{\alpha_1 \alpha'_1} \\ &\quad \cdot \delta_{\alpha_2 \alpha'_2} \delta_{\alpha_3 \alpha'_3} \delta_{\alpha_4 \alpha'_4}]_{\text{sym}(\alpha_1 \alpha_2 \alpha_3 \alpha_4)} \\ &\quad + g[(\beta \gamma^5)_{\alpha_1 \alpha'_1} \gamma_{\alpha_2 \alpha'_2}^5]_{\text{sym}(\alpha'_1 \alpha'_2)} \delta_{\alpha_3 \alpha'_3} \delta_{\alpha_4 \alpha'_4} \\ &\quad + g\delta_{\alpha_1 \alpha'_1} \delta_{\alpha_2 \alpha'_2} [(\beta \gamma^5)_{\alpha_3 \alpha'_3} \gamma_{\alpha_4 \alpha'_4}^5]_{\text{sym}(\alpha_3 \alpha_4)}\} \\ &\quad \cdot d_{\alpha'_1 \alpha'_2 \alpha'_3 \alpha'_4}^{tt'}(\mathbf{r}) d\mathbf{r}. \end{aligned} \quad (3.6)$$

The corresponding energy eigenvalue equation for free gravitons then reads

$$E |\hat{\mathcal{G}}\gamma^f = \mathcal{H}_B^0 |\hat{\mathcal{G}}\gamma^f. \quad (3.7)$$

As can be easily seen, in (3.7) the superspin-isospin dependence drops out, i.e., our effective equations for the determination of the free graviton states give no hint how to choose these states with respect to the set $\{T^t \otimes T^{t'}\}$. This dependence can, however, be deduced from a consideration of the BBW-states. According to Rodriguez and Lorente [60] for massless gravitons with the special momentum $p = (E, 0, 0, E)$ only the local amplitudes with $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha$ give a non-vanishing contribution to the graviton states. If this condition is transferred to the microscopic theory it means that for $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha$ the limes $\mathbf{r}_i \rightarrow \mathbf{r}$, $i = 1, 2, 3, 4$ of (3.4) for all combinations of auxiliary fields must exist. In particular for the original field strengths of (1.1) we have $\phi_1 = \sum_{ii'} \phi^{ii'}$

and for these fields the limes of (3.4) reads

$$\begin{aligned} \lim_{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4 \rightarrow \mathbf{r}} C_n &= e^{i\mathbf{K}\mathbf{r}} \{\phi_1(0)^2 \hat{\phi}(0) S_{\alpha\alpha}^\sigma S_{\alpha\alpha}^{\sigma'} T_{\alpha_1 \alpha_2}^t T_{\alpha_3 \alpha_4}^{t'}\}_{\text{as}}, \end{aligned} \quad (3.8)$$

i.e., in this expression antisymmetrization rests only on the $T^t \otimes T^{t'}$ -product. The only combination for any $T^t \otimes T^{t'}$ -product that survives this antisymmetrization is

$$\{T_{\alpha_1 \alpha_2}^t T_{\alpha_3 \alpha_4}^{t'}\}_{\text{as}} = \{\delta_{\alpha_1 \alpha_1} \delta_{\alpha_2 \alpha_2} \delta_{\alpha_3 \alpha_3} \delta_{\alpha_4 \alpha_4}\}_{\text{as}} \quad \forall t, t'. \quad (3.9)$$

Thus by this limes condition the superspin-isospin dependence is uniquely determined. Furthermore, it will be demonstrated that in all relevant interactions only $t = t'$ is admitted and that the coupling constant is independent of t . In accordance with (3.9) we are thus allowed to choose an arbitrary product $T^t \otimes T^t$ as a representative of (3.9). In the

following we use $T^t \equiv T = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}$ and omit the t, t' indexing.

Due to the special structure of (3.6), Eq. (3.7) can be interpreted as the functional energy representation of a local field theory with local field operators $\psi_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}(x)$. If we represent $|\hat{\mathcal{G}}\gamma^f$ by the connected functional defined by (2.10) (2.11) and substitute this in (3.7) we obtain the linear part of (2.15) which reads

$$\begin{aligned} [i\gamma^\mu \partial_\mu - M - g]_{\alpha_1 \alpha'_1} \psi_{\alpha'_1 \alpha_2 \alpha_3 \alpha_4}(x) &= -g \gamma_{\alpha_1 \alpha'_1}^5 \gamma_{\alpha_2 \alpha'_2}^5 \psi_{\alpha'_1 \alpha'_2 \alpha_3 \alpha_4}(x), \\ [i\gamma^\mu \partial_\mu - M - g]_{\alpha_2 \alpha'_2} \psi_{\alpha_1 \alpha'_2 \alpha_3 \alpha_4}(x) &= -g \gamma_{\alpha_1 \alpha'_1}^5 \gamma_{\alpha_2 \alpha'_2}^5 \psi_{\alpha_1 \alpha'_2 \alpha_3 \alpha_4}(x), \end{aligned} \quad (3.10)$$

and equations with exchange of 1 2 \rightarrow 3 4. In particular for linear fields the solutions of the functional equation and of the corresponding classical equations coincide exactly as is the case with (3.7) and (3.10).

Equations (3.10) are generalized BBW-equations. The advantage of having derived generalized and not the original BBW-equations will soon become obvious. The expansion of the multispinor reads

$$\begin{aligned} \psi_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} &= (\gamma^a C)_{\alpha_1 \alpha_2} (\gamma^b C)_{\alpha_3 \alpha_4} \psi_{a, b} \\ &\quad + (\gamma^a C)_{\alpha_1 \alpha_2} (\Sigma^{a'b'} C)_{\alpha_3 \alpha_4} \psi_{a, a'b'} \\ &\quad + (\Sigma^{ab} C)_{\alpha_1 \alpha_2} (\gamma^{a'} C)_{\alpha_3 \alpha_4} \psi_{ab, a'} \\ &\quad + (\Sigma^{ab} C)_{\alpha_1 \alpha_2} (\Sigma^{a'b'} C)_{\alpha_3 \alpha_4} \psi_{ab, a'b'}. \end{aligned} \quad (3.11)$$

In contrast to the solutions of the BBW-equations the multispinors (3.11) need not be fully symmetrized since these spinor amplitudes are derived from the fully antisymmetric states (3.2) and are not forced to be completely symmetric. By substitution of (3.11) into (3.10) it can be shown that free graviton states result only for $\psi_{a,a'} \equiv 0$. This assumption is compatible with the general equations resulting from (3.10). For brevity we do not discuss this in length. For $\psi_{a,a'} \equiv 0$ we obtain from (3.10) with $g = -M$ and rescaling $\psi_{ab,a'b'}$ by $(2M)^{-1}$

$$\begin{aligned}\psi_{ab,a'b'}(x) &= \partial_{[a}\psi_{b],a'b'}(x) + \partial_{[a'}\psi_{ab,b']}(x), \\ \partial_\mu\psi_{a\mu,a'b'}(x) &= 0, \\ \partial_\mu\psi_{ab,a'\mu}(x) &= 0, \\ \partial_\mu\psi_{a,a'\mu}(x) + \partial_\mu\psi_{\mu a,a'}(x) &= 0.\end{aligned}\quad (3.12)$$

This system is solved by the ansatz

$$\begin{aligned}\psi_{ab,a'b'}(x) &= \partial_a\partial_{a'}\chi_{bb'} - \partial_a\partial_{b'}\chi_{aa'} \\ &\quad - \partial_b\partial_{a'}\chi_{b'a} + \partial_b\partial_{b'}\chi_{aa'}, \\ \psi_{a,a'b'}(x) &= \partial_{a'}\chi_{ab'} - \partial_{b'}\chi_{aa'} + \partial_a\hat{\chi}_{a'b'}, \\ \psi_{ab,a'}(x) &= \partial_a\chi_{ba'} - \partial_b\chi_{aa'} + \partial_a\hat{\chi}_{ab}\end{aligned}\quad (3.13)$$

with $\chi_{ab} = \chi_{ba}$ symmetric and $\hat{\chi}_{ab} = -\hat{\chi}_{ba}$ anti-symmetric functions. These functions have to satisfy the homogeneous equations

$$\begin{aligned}\partial_\mu\partial^\mu\chi_{aa'} - \partial^\mu\partial_a\chi_{a'\mu} &= 0, \\ \partial^\mu\partial_{a'}\chi_{a\mu} &= \partial^\mu\partial_a\chi_{a'\mu},\end{aligned}\quad (3.14)$$

$$\text{and } \partial^\mu\hat{\chi}_{\mu a'} = 0. \quad (3.15)$$

Remembering our initial hypothesis about the formation of graviton states, we see that for $\chi_{aa'} = A_a A_{a'}$ and $\hat{\chi}_{ab} \equiv 0$ the solutions (3.13) are indeed the direct product of vectorpotentials and corresponding fields resp. But for the further interpretation of (3.13) the relation to geometric quantities has to be established.

The solutions (3.13) for $\psi_{ab,a'b'}$ coincide with those of the corresponding BBW-equation. They are interpreted as the components of the Riemann or Weyl curvature tensor, resp., in the weak field approximation, [59–61]. The solutions (3.13) for $\psi_{a,a'b'}$ and $\psi_{ab,a'}$ differ from those of the corresponding BBW-equation, because for the latter

in the case of massless gravitons $\psi_{a,a'b'}$ and $\psi_{a'b',a}$ vanish identically, [60], whereas in our case these quantities do not vanish! This difference is *essential* for obtaining a meaningful theory of gravity. It is reasonable to interpret $\psi_{ab,a'}$ and $\psi_{a,a'b'}$ as spinorial affine connections and if these quantities vanish an essential geometrical quantity is not available. It is this fact that caused earlier approaches to gravity based on BBW-equations to fail. An extensive geometrical interpretation of our approach will be given in Section 5.

4. Effective Nonlinear Graviton Equations

To evaluate the complete Eq. (2.15) we will use the graviton states of Section 3. However, before doing any special calculation we have to discuss selfconsistency carefully. As the linear states are completely fixed by the linear theory they cannot be compatible with the nonlinear theory. In the perturbation theoretic approach to gravity [65] this contradiction is ignored. In this approach we do not use perturbation theory and thus solve this problem explicitly. To reconcile the nonlinear equations (2.15) with their linear already calculated part (3.10) we must somehow relax the linear state representation (3.13). This can simply be done by abandoning the wave equations (3.14) and (3.15) for χ_{ab} and $\hat{\chi}_{ab}$ respectively. But even if χ_{ab} and $\hat{\chi}_{ab}$ can be varied freely this is not sufficient for selfconsistency. This can be concluded from the physical interpretation of χ_{ab} and $\hat{\chi}_{ab}$. In Sect. 5 we will show that χ_{ab} represents genuine deviations of the metric g_{ab} from the Minkowski metric η_{ab} in orthonormal (tetrad) coordinates, while $\hat{\chi}_{ab}$ represents local Lorentz transformations. Now, according to a theorem of Lichnerowicz [100] the homogeneous vacuum equations of Einstein gravity $R_{ik} = 0$ admit only the Minkowski metric as a regular solution. Thus, if our homogeneous equation (2.15) is to correspond to Einstein gravity no nontrivial deviation χ_{ab} from the Minkowski metric η_{ab} can be compatible with (2.15). Therefore we see that the compatibility of the linear theory with the nonlinear equation (2.15) requires the exclusion of the symmetric part χ_{ab} of (3.13). By selfconsistency we are thus restricted to the discussion of the effects of local Lorentz-transformations. Therefore, by selfconsistency we ar-

rive at the invariance arguments of Poincaré-gauge theories and of Weinberg's derivation of gravity. In the following we will thus treat (2.15) in accordance with this program.

Turning now to the evaluation of (2.15), we see that apart from the undressed graviton states of Sect. 3, the selfinteraction (2.6) also contains a contribution of the graviton polarization cloud. This state has first to be computed before any other calculation can be carried out.

The polarization cloud part of (2.6) is given by its dual state representation. However, it suffices to calculate the original polarization cloud states because their dual counterpart states can be obtained by simply replacing all subcluster states by their dual counterparts.

To compute the original polarization cloud states it is convenient to start with the time-ordered states. For $t_1 < t_2 < \dots < t_6$ we have

$$\begin{aligned} T_{I_1 \dots I_6}^m &:= \langle 0 | \varphi_{I_1} \varphi_{I_2} \varphi_{I_3} \varphi_{I_4} \varphi_{I_5} \varphi_{I_6} | m \rangle \\ &= \sum_k \langle 0 | \varphi_{I_1} \varphi_{I_2} \varphi_{I_3} | k \rangle \langle k | \varphi_{I_4} \varphi_{I_5} \varphi_{I_6} | m \rangle \end{aligned} \quad (4.1)$$

with $\{|k\rangle\}$ as intermediate states. As far as preons are concerned in the lowest approximation $\langle k | = \langle 0 | \varphi(k)$ holds where $\varphi(k)$ is that part of φ_I which generates a preon state $\langle k |$. In this approximation no other fermions contribute to this decomposition and we thus obtain from (4.1)

$$\begin{aligned} T_{I_1 \dots I_6}^m &= \sum_k \langle 0 | \varphi_{I_1} \varphi_{I_2} \varphi_{I_3} | k \rangle \\ &\cdot \langle 0 | \varphi(k) \varphi_{I_4} \varphi_{I_5} \varphi_{I_6} | m \rangle. \end{aligned} \quad (4.2)$$

Formula (4.2) is the only approximation where the original graviton function emerges, surrounded by a polarization cloud which is due to the dressing of one of its fermionic constituents. We thus assume to have derived the correct polarization cloud state for a graviton state (in lowest order).

The transition to normalordered states can simply be performed by omitting all two-point-function contributions to (4.2) and by subsequent antisymmetrization. This gives

$$\begin{aligned} C_{I_1 \dots I_6}^m &:= \left\{ \sum_k \langle 0 | N \varphi_{I_1} \varphi_{I_2} \varphi_{I_3} | k \rangle \right. \\ &\cdot \left. \langle 0 | N \varphi(k) \varphi_{I_4} \varphi_{I_5} \varphi_{I_6} | m \rangle \right\}_{\text{as}}. \end{aligned} \quad (4.3)$$

For these functions the transition to equal times can be performed and for equal times the functions $\langle 0 | N \varphi_{I_1} \varphi_{I_2} \varphi_{I_3} | k \rangle$ are known in their general form from [76]. We substitute these functions into (4.3) and after some elementary rearrangements we approximately obtain for fairly concentrated states and $t_1 = \dots = t_6$

$$\begin{aligned} C_{I_1 \dots I_6}^m &\approx \{ U^{i_1 i_2} Z_{\alpha_1 \alpha_2}^{\kappa_1 \kappa_2} g(\mathbf{r}_1 - \mathbf{r}_2) \\ &\cdot h(\mathbf{r}_3 - \tfrac{1}{2}(\mathbf{r}_2 + \mathbf{r}_1)) C_{I_3 I_4 I_5 I_6}^m \}_{\text{as}}. \end{aligned} \quad (4.4)$$

The transition to the corresponding dual states can be performed by replacing the subcluster functions of (4.4) by their dual states. With $I =$ complement to $I_1 I_2$ this leads to

$$R_{I_1 I_2}^m := \{ P_{I_1 I_2}(I) R_{I_3 I_4 I_5 I_6}^m \}_{\text{as}}, \quad (4.5)$$

where

$$\begin{aligned} P_{I_1 I_2}(I) &:= S^{i_1 i_2} Z_{\alpha_1 \alpha_2}^{\kappa_1 \kappa_2} \sigma_3(\mathbf{r}_1 - \mathbf{r}_2) \\ &\cdot f(\mathbf{R} - \tfrac{1}{2}(\mathbf{r}_2 + \mathbf{r}_1)) \end{aligned} \quad (4.6)$$

with $Z := \gamma_5 \otimes C$ and $\mathbf{R} = (\mathbf{r}_3 + \mathbf{r}_4 + \mathbf{r}_5 + \mathbf{r}_6)1/4 \approx \mathbf{r}_3$ for highly concentrated graviton states.

The wave functions (4.6) can be interpreted as boson states with internal structure described by σ_3 being bound to a center \mathbf{R} by bound state functions f . In preceding papers [75, 78, 79] it was shown that the one-time functions fulfill Schrödinger-like equations. From these experiences we can draw some general conclusions: For small boson masses m the functions σ_3 are highly concentrated because small masses correspond to strong binding. If such light bosons appear as constituents of the polarization cloud of preons their contribution is practically suppressed due to large resonance denominators of order of the preon mass. Thus only heavy bosons can essentially contribute to the polarization cloud of a preon. But in this case σ_3 is wide-spread because heavy masses correspond to weak binding. For heavy bosons, however, the binding to the origin \mathbf{R} is strong. Summarizing these considerations we can conclude that the polarization cloud contributions (4.6) in the leading term approximation can only contain heavy bosons with wide spread σ_3 and narrow f . In particular, we can assume that σ_3 is orthogonal on the light vector boson state functions φ_1 which appear in the graviton states themselves.

Naturally, all these considerations need an improved justification by more detailed calcula-

tions. For a first draft of a rather complicated theory it is, however, impossible to justify all in detail, because a precondition for carrying out detailed calculations is a clear idea about the physical background that governs these calculations. In this way we assume that the preliminary discussion of polarization cloud contributions offers this possibility and we apply the results of this discussion to the evaluation of (2.13).

Substitution of (4.5) into (2.13) and straightforward rearrangements by using symmetry and anti-symmetry properties lead to the following expression

$$\begin{aligned} \mathcal{H}_B^I = & 12 \sum_{\substack{L_1 L_2 L_3 \\ I_1 \dots I_6 \\ m n n'}} R_{I_1 I_2 I_3 I_4}^m P_{I_5 I_6} \\ & \cdot \{ -4 \hat{W}_{I_5 L_1 L_2 L_3} C_n^{L_3 I_4 I_1 I_2} C_n^{L_1 L_2 I_3 I_4} \\ & + 2 \hat{W}_{I_5 L_1 L_2 L_3} C_n^{L_3 I_1 I_4 I_6} C_n^{L_1 L_2 I_2 I_3} \\ & - 5 \hat{W}_{I_1 L_1 L_2 L_3} C_n^{L_3 I_2 I_4 I_6} C_n^{L_1 L_2 I_3 I_5} \} b_m d_n d_n . \end{aligned} \quad (4.7)$$

If the wave functions in (4.7) are correctly antisymmetrized and the antisymmetry of the vertex is taken into account, each term of (4.7) formally contains $3(4!)^3$ integrals. A number which would prevent any evaluation. Due to the formation of gravitons by two vector boson states, the graviton wave functions (3.4), however, possess such a high structural symmetry that the number of independent integrals is considerably reduced. Formally, we can write for C_n of (3.4)

$$C_n = e^{iKR} \{ Z_{12} Z'_{34} u(12, 34) \}_{\text{as}(1,2,3,4)}, \quad (4.8)$$

where $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4)1/4$ and Z and u follow by comparison with (3.4). If antisymmetrization is explicitly performed and the symmetry properties of u and Z are taken into account, we obtain from (4.8)

$$\begin{aligned} C_n = & \{ [Z_{12} Z'_{34}]_{\text{sym}} u(12, 34) \\ & + [Z_{14} Z'_{23}] u(14, 23) \\ & + [Z_{31} Z'_{24}] u(13, 14) \} e^{iKR} . \end{aligned} \quad (4.9)$$

With this special form of the wave functions each term of (4.7) eventually contains only 27 independent integrals. These integrals were individually estimated along the lines which were extensively discussed in [75, 76]. So here we will not repeat

explicit calculations, but give only the results. If the properties of the polarization cloud wave functions (4.6) and of the original graviton wave functions (4.9) are observed, in the leading term approximation only one integral of (4.7) is left. By using the definitions (3.5) and $S^\sigma := (s^\sigma C)$ and integrating over the internal coordinates of the wave functions $\{C_n\}$ we obtain the following expression for this term

$$\begin{aligned} \mathcal{H}_B^I = & k \sum_{mn'} \int R \left(\begin{array}{cccc} r_1 & r_2 & r_3 & r_4 \\ i_1 & i_2 & i_3 & i_4 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \kappa_1 & \kappa_2 & \kappa_3 & \kappa_4 \end{array} \middle| m \right) \\ & \cdot [s^{\sigma'} \gamma^0 s^\sigma - s^{\sigma'} \gamma^0 \gamma^5 s^\sigma \gamma^5]_{\alpha_3 \beta_3} \\ & \cdot C \left(\begin{array}{cccc} r_1 & r_2 & r_3 & r_4 \\ i_1 & i_2 & i_3 & i_4 \\ \alpha_1 & \alpha_2 & \beta_3 & \alpha_4 \\ \kappa_1 & \kappa_2 & \kappa_3 & \kappa_4 \end{array} \middle| n' \right) \\ & \cdot b_m d_n d_{\sigma, \sigma'}(\mathbf{r}_3) d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 , \end{aligned} \quad (4.10)$$

where according to Sect. 3 for C_n the superspin-isospin combination $T^I = T^{I'} = T$ has been used. The factor k is a numerical constant whose value follows from the integral.

Formula (4.10) can still be simplified. According to Sect. 2 we first transform (4.10) to connected functionals by the substitution $d_n \rightarrow z_n$. Then we observe that $z_{\sigma, \sigma'} = z_{ab, a'b'}$ and $z_{ab, a'}$ drop out from the beginning and that due to (3.13) we have $z_{a, a'} \equiv 0$. So the only term that is left in (4.10) is $z_{\sigma, \sigma'} = z_{a, a'b'}$. Furthermore, (4.10) admits a symmetrization and if we introduce the abbreviation

$$X_i \equiv X(i) = \delta_{\alpha_1 \alpha'_1} \dots X_{\alpha_i \alpha'_i} \dots \delta_{\alpha_4 \alpha'_4} \quad (4.11)$$

we can eventually write (4.10) in the abridged form

$$\begin{aligned} \mathcal{H}_B^I = & k \sum_{mn} \left\langle R_m \left[\sum_{i=1}^4 (\Sigma^{a'b'} \alpha^a)_i z_{a, a'b'}(\mathbf{r}_i) \right] C_n \right\rangle b_m z_n . \end{aligned} \quad (4.12)$$

Owing to the compatibility condition initially discussed, the function $z_{a, a'b'}(\mathbf{r})$ can only have the form

$$z_{a, a'b'}(\mathbf{r}) = \partial_a \hat{\chi}_{a'b'}(\mathbf{r}) \quad (4.13)$$

with $\hat{\chi}$ antisymmetric. Defining then the spin tensor

$$\hat{Z}(\mathbf{r}) := k \sum_{a'b'} [\Sigma^{a'b'} \hat{\chi}_{a'b'}(\mathbf{r})] \quad (4.14)$$

we obtain for (4.12) with (4.14) the expression

$$\mathcal{H}_B^I = \sum_{mna} \left\langle R_m \left[\sum_{i=1}^4 \partial_a^i \hat{Z}(\mathbf{r}_i, i) \alpha^a(i) \right] C_n \right\rangle b_m z_n \quad (4.15)$$

This expression can be combined with the kinetic energy term \mathcal{H}_B^k of (2.12) to give the formula

$$\mathcal{H}_B^k + \mathcal{H}_B^I = \sum_{mna} \left\langle R_m \left\{ \sum_{i=1}^4 [\partial_a^i + \partial_a^i \hat{Z}(i)] \alpha^a(i) \right\} C_n \right\rangle b_m z_n \quad (4.16)$$

for connected functionals. The term (4.16) allows a rearrangement to the following form

$$\mathcal{H}_B^k + \mathcal{H}_B^I = \sum_{mna} \left\langle R_m e^{-\sum_l \hat{Z}(\mathbf{r}_l, l)} \sum_{i=1}^4 \partial_a^i e^{\sum_h \hat{Z}(\mathbf{r}_h, h)} \alpha^a(i) C_n \right\rangle b_m z_n \quad (4.17)$$

Subsequent integration over the internal coordinates of the wave functions R_m and C_n enforces the substitution of \mathbf{r}_i and ∂^i by the corresponding center of mass coordinates \mathbf{r} and ∂ due to the internal wave functions strongly concentrated about \mathbf{r} . If afterwards the exponentials are eliminated, (4.17) goes over into

$$\mathcal{H}_B^k + \mathcal{H}_B^I = \int b(\mathbf{r}) \left\{ \sum_{i=1}^4 \left[\partial_a + \sum_{h=1}^4 \partial_a \hat{Z}(\mathbf{r}, h) \right] \alpha^a(i) \right\} z(\mathbf{r}) d\mathbf{r} \quad (4.18)$$

with $z(\mathbf{r}) = z_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}(\mathbf{r})$ and $b(\mathbf{r}) = b_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}(\mathbf{r})$ in accordance with (3.5). In the last step we collect all terms which contribute to \mathcal{H}_B and we eventually obtain for the Eq. (2.15) the explicit expression

$$\int b(\mathbf{r}) \left\{ \sum_{i=1}^4 \left[\partial_a + \sum_{h=1}^4 k (\Sigma^{a'b'}(h) z_{a,a'b'}(\mathbf{r})) \right] \alpha^a(i) \right. \\ \left. + \frac{\partial}{\partial t} + g[(\beta \gamma^5)_1 (\gamma^5)_2 + (\beta \gamma^5)_3 (\gamma^5)_4] \right\} z(\mathbf{r}) d\mathbf{r} = 0 \quad (4.19)$$

where in particular the results of Sect. 2 were used.

5. Tetrad Equations of Gravity

Equation (4.19) is the intended nonlinear graviton equation. It holds for arbitrary $b(\mathbf{r}) \equiv b(\mathbf{r}, 0)$. Since in the energy-representation the center of mass time t can be fixed at an arbitrary value we consider Eq. (4.19) for $b(\mathbf{r}, t)$ and $z(\mathbf{r}, t)$. Then (4.19) is equivalent to the system of equations

$$(\alpha^k \partial_\beta - M\beta - g\beta)_{\alpha_1 \alpha'_1 \alpha'_2 \alpha_3 \alpha_4}(x) \\ + \frac{\partial}{\partial t} z_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}(x) + g(\beta \gamma^5)_{\alpha_1 \alpha'_1 \gamma^5 \alpha_2 \alpha'_2 \alpha_3 \alpha_4}(x) \\ + k \sum_{h=1}^4 (\Sigma^{a'b'}(h)) z_{a,a'b'}(x) \alpha_{\alpha_1 \alpha'_1}^a z_{\alpha'_2 \alpha_3 \alpha_4}(x) = 0 \quad (5.1)$$

and equations which arise through cyclic permutation of the spinorial indices.

The further evaluation of (5.1) rests on the physical interpretation of the functions $z(x)$. Since we work with spinorial quantities, the corresponding basic geometrical quantities are the tetrads $\{e_i^a\}$ [3], which diagonalize the metric tensor g_{ij} by definition, i.e., we have $g_{ij} = e_i^\alpha e_j^\beta \eta_{\alpha\beta}$. The tetrads are not uniquely fixed. Having diagonalized g_{ij} by a certain set $\{e_i^a\}$ we can subsequently perform a Lorentztransformation and obtain a new set $\{e_i^a\}$ doing the same. For small deviations of g_{ij} from η_{ij} and for infinitesimal Lorentztransformations these two effects can be separated and we obtain $e_{i\alpha} = e_i^\alpha \eta_{\alpha\beta} = \eta_{i\alpha} + a_{i\alpha} + s_{i\alpha}$, where the antisymmetric part $a_{i\alpha}$ corresponds to infinitesimal Lorentztransformations, while the symmetric part reflects the deviation of g_{ij} from η_{ij} . In order to satisfy the self-consistency condition for homogeneous graviton equations which was discussed in Sect. 4, we are only allowed to admit Lorentztransformations and due to their group structure we can restrict ourselves to infinitesimal Lorentztransformations, i.e., we use $e_{i\alpha} = \eta_{i\alpha} + a_{i\alpha}$. For such transformations the spinorial affine connection [7, 8] is given by

$$\Gamma_{\alpha, \beta\gamma} = \partial_\alpha a_{\beta\gamma} \quad (5.2)$$

and a comparison with (3.13) shows that $\psi_{a,a'b'}$ and $z_{a,a'b'}$ have to be identified with $\Gamma_{a,a'b'}$. In particular, it follows from (3.13) that the antisymmetric part $\hat{\chi}_{ab} \equiv a_{ab}$ depends on x , i.e., the graviton theory possesses solutions which rigorously reproduce the spinorial affine connections for local infinitesimal Lorentztransformations. In the linear theory such local transformations are subjected to the condition (3.15), in the nonlinear theory we relax this condition. Nevertheless, also in the nonlinear theory an additional condition has to be imposed on these local transformations in order to avoid symmetry breaking.

Equations (5.1) have a structural resemblance to Hartree-Fock equations. If one starts with a rotational forminvariant many-particle Hamilton operator, its corresponding Hartree-Fock equations in general do break this symmetry. The original symmetry is only preserved if one solely admits rotationally forminvariant one-particle functions, i.e., one-dimensional representations of the rotation group for the calculation of the Hartree-Fock potentials. For all other configurations symmetry is broken. With respect to Eqs. (5.1) we expect them to be Poincaré- or Lorentzforminvariant resp., since the initial spinor preon Eqs. (1.1) have this property. Transferring the Hartree-Fock experience to our case in order to avoid relativistic symmetry breaking, we are thus forced to use in the energy-representation forminvariant representations for the potential with respect to the little group which in our case is the rotation group. Obviously, in (5.1) the function $z_{a,a'b'}(x)$ plays the role of the “potential” and thus $z_{a,a'b'}(x)$ has to be forminvariant under the little group operations. Since $z_{a,a'b'}(x)$ is given by

$$z_{a,a'b'}(x) = \partial_a \hat{\chi}_{a'b'} \quad (5.3)$$

i.e., by (3.13) with $\chi_{ab} \equiv 0$, the antisymmetric function $\hat{\chi}_{a'b'}$ must satisfy the little group form-invariance condition. For infinitesimal local Lorentztransformations the most general form of $\hat{\chi}_{ab}$ reads

$$\begin{aligned} \hat{\chi}_{ab}(x) = & \sum_{k=1}^3 \varepsilon_k(x) M_{aa'}^{k0} \eta_{a'b'} \\ & + \sum_{klm=1}^3 \hat{\varepsilon}_k(x) \varepsilon_{klm} M_{ab}^{lm}, \end{aligned} \quad (5.4)$$

where $\{M^{ij}\}$ are the infinitesimal generators of the homogeneous Lorentzgroup and $\{\varepsilon_k(x), \hat{\varepsilon}_k(x)\}$ their local infinitesimal parameters. Under the little group these parameters transform as three-vectors. Thus in order to keep (5.4) form invariant with respect to the little group we can assume

$$\varepsilon_k(x) = \partial_k \varphi(x); \quad \hat{\varepsilon}_k(x) = \partial_k \hat{\varphi}(x), \quad (5.5)$$

where $\varphi(x)$ and $\hat{\varphi}(x)$ are two arbitrary (but small) scalar functions.

The substitution of the little group form-invariant $z_{a,a'b'}(x)$ into (5.1) allows the commutation of α^a with $\Sigma^{a'b'} z_{a,a'b'}$. Therefore we can rewrite (5.1) into the fully relativistically invariant form

$$\begin{aligned} (i\gamma^\mu \partial_\mu - M - g)_{\alpha_1 \alpha'_1} z_{\alpha'_1 \alpha_2 \alpha_3 \alpha_4}(x) \\ + g \gamma_{\alpha_1 \alpha'_1}^5 \gamma_{\alpha_2 \alpha'_2}^5 z_{\alpha'_1 \alpha'_2 \alpha_3 \alpha_4}(x) \\ + k \sum_{h=1}^4 \gamma_{\alpha_1 \alpha'_1}^a (\Sigma^{a'b'})_{\alpha_h \alpha'_h} z_{a,a'b'}(x) z_{\alpha'_1 \dots \alpha'_h \dots \alpha_4}(x) \\ = 0. \end{aligned} \quad (5.6)$$

In analogy to the linear case (3.11) we apply the expansion

$$\begin{aligned} z_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}(x) = & (\gamma^a C)_{\alpha_1 \alpha_2} (\Sigma^{a'b'} C)_{\alpha_3 \alpha_4} z_{a,a'b'}(x) \\ & + (\Sigma^{ab} C)_{\alpha_1 \alpha_2} (\gamma^{a'} C)_{\alpha_3 \alpha_4} z_{ab,a'}(x) \\ & + (\Sigma^{ab} C)_{\alpha_1 \alpha_2} (\Sigma^{a'b'} C)_{\alpha_3 \alpha_4} z_{ab,a'b'}(x) \end{aligned} \quad (5.7)$$

with $z_{a,a'b'}(x) = z_{a'b',a}(x)$. Then $z_{ab,a'b'}(x)$ and $z_{a,a'b'}(x)$ are to be considered as anholonomic representations of the curvature tensor and the affine connection resp. Direct evaluation of (5.6) by means of (5.7) leads to the equations

$$\begin{aligned} \partial_\mu z_{\mu\nu,cd} - z_{\mu,\mu\delta} z_{\delta\nu,cd} - z_{\mu,\nu\delta} z_{\mu\delta,cd} \\ - z_{\mu,cd} z_{\mu\nu,\delta d} - z_{\mu,d\delta} z_{\mu\nu,c\delta} = 0 \end{aligned} \quad (5.8)$$

and

$$z_{\mu\nu,cd} = 2 \partial_{[\mu} z_{\nu],cd} + 2 z_{[\mu|c\varrho} z_{\nu],d\varrho} - 2 z_{[\mu,\nu] \varrho} z_{\varrho,cd} \quad (5.9)$$

if we put $g = -M$ and remove the factors M and k by rescaling. Furthermore, from symmetry considerations it follows

$$z_{\mu\nu,\mu d} = 0. \quad (5.10)$$

These equations are the tetrad version of Einstein's homogeneous vacuum equations, see Edgar [101].

- [1] P. G. O. Freund, *Supersymmetry*, Cambridge University Press, 1986.
- [2] M. I. Duff, *Supergravity*, Kaluza-Klein and Superstrings, CERN-TH. 4568/86.
- [3] H. Weyl, *Ann. Phys. (Germany)* **59**, 101 (1919); *Z. Phys.* **56**, 330 (1929).
- [4] C. N. Yang and R. L. Mills, *Phys. Rev.* **96**, 191 (1954).
- [5] R. Utiyama, *Phys. Rev.* **101**, 1597 (1956).
- [6] T. W. B. Kibble, *J. Math. Phys.* **2**, 212 (1961).
- [7] F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester, *Rev. Mod. Phys.* **48**, 393 (1976).
- [8] F. W. Hehl, J. Nitsch, and P. von der Heyde, *Gen. Rel. Grav.* **1**, 329 (1980).
- [9] M. O. Katanaev, *Theor. Math. Phys.* **54**, 248 (1983).
- [10] F. Müller-Hoissen, *Ann. Inst. Henri Poincaré* **40**, 21 (1984).
- [11] M. H. Friedman, *Phys. Rev. D* **12**, 2528 (1975).
- [12] F. Mansouri and L. N. Chang, *Phys. Rev. D* **13**, 3192 (1976).
- [13] L. N. Chang and F. Mansouri, *Phys. Rev. D* **17**, 3168 (1978).
- [14] A. Trautman, *General Relativity and Gravitation*, Vol. 1, ed. by A. Held, Plenum Press, New York/London, 1980, p. 287.
- [15] F. G. Basombrio, *Gen. Rel. Grav.* **12**, 109 (1980).
- [16] E. A. Ivanov and J. Niederle, *Phys. Rev. D* **25**, 976 (1982).
- [17] W. Pauli, *Phys. Z.* **20**, 457 (1919).
- [18] C. Lanczos, *Rev. Mod. Phys.* **21**, 497 (1949); **29**, 337 (1957).
- [19] G. Stephenson, *Nuovo Cim.* **9**, 263 (1958).
- [20] M. Carmeli and S. I. Fickler, *Phys. Rev. D* **5**, 290 (1972).
- [21] M. Carmeli, *J. Math. Phys.* **11**, 2728 (1970); *Nucl. Phys. B* **38**, 621 (1972); *Gen. Rel. Grav.* **5**, 287 (1974); *Phys. Rev. Lett.* **36**, 59 (1976).
- [22] P. G. O. Freund, *Ann. Phys. New York* **84**, 440 (1974).
- [23] C. N. Yang, *Phys. Rev. Lett.* **33**, 445 (1974).
- [24] A. H. Thompson, *Phys. Rev. Lett.* **34**, 507 (1975).
- [25] E. E. Fairchild, *Phys. Rev. D* **14**, 384 (1976).
- [26] C. A. Lopez, *Int. J. Theor. Phys.* **16**, 167 (1977).
- [27] L. Smolin, *Nucl. Phys. B* **160**, 253 (1979).
- [28] B. Hasslacher and E. Mottola, *Phys. Lett.* **99B**, 221 (1981).
- [29] D. G. Boulware, G. T. Horowitz, and A. Strominger, *Phys. Rev. Lett.* **50**, 1726 (1983).
- [30] H. R. Pagels, *Phys. Rev. D* **29**, 1690 (1984).
- [31] M. Kaku, *Nucl. Phys. B* **203**, 285 (1982).
- [32] H. Dehnen and F. Ghaboussi, *Nucl. Phys. B* **262**, 144 (1985).
- [33] I. Bars and S. W. MacDowell, *Phys. Lett.* **71b**, 111 (1977).
- [34] D. Amati and G. Veneziano, *Phys. Lett.* **105B**, 358 (1981); *Nucl. Phys. B* **204**, 451 (1982).
- [35] A. Sen, *Phys. Lett.* **119b**, 89 (1982).
- [36] A. D'Adda, *Phys. Lett.* **119B**, 334 (1982).
- [37] H. Saller, *Nuovo Cim.* **91B**, 157 (1986).
- [38] N. Sanchez, *Phys. Rev. D* **26**, 2589 (1982); *Phys. Lett.* **144B**, 217 (1984); CERN-Report TH. 4583 (1986).
- [39] R. Percacci, *Gen. Rel. Grav.* **14**, 1043 (1982).
- [40] P. Jordan, *Ergebn. Exakt. Naturwiss.* **7**, 158 (1928).
- [41] L. de Broglie, *C. R. Acad. Sci. Paris* **195**, 862 (1932).
- [42] P. Jordan, *Z. Physik* **93**, 464 (1935).
- [43] R. Kronig, *Physica* **3**, 1120 (1936).
- [44] L. de Broglie, *Théorie générale des particules à spin*, Gauthier-Villars, Paris 1943.
- [45] A. Proca, *J. Physique Radium* **7**, 347 (1937); **9**, 61 (1938).
- [46] N. Kemmer, *Proc. Roy. Soc. London A* **173**, 91 (1940).
- [47] M. A. Tonnelat, *C. R. Acad. Sci. Paris* **212**, 263, 384, 430, 687 (1941); **214**, 253 (1942).
- [48] G. Petiau, *C. R. Acad. Sci. Paris* **212**, 47, 292, 1126 (1941); **214**, 610 (1942); **215**, 77 (1942).
- [49] P. M. Dirac, *Proc. Roy. Soc. London A* **155**, 447 (1936).
- [50] M. Fierz, *Helv. Phys. Acta* **12**, 3 (1939).
- [51] W. Pauli and M. Fierz, *Helv. Phys. Acta* **12**, 297 (1939).
- [52] V. Bargmann and E. P. Wigner, *Proc. Nat. Acad. Sci. USA* **34**, 211 (1948).
- [53] F. Bopp, *Bayr. Acad. Wissensch. Naturw. Klasse S. B.* 1958, p. 167.
- [54] W. Heisenberg, *Rev. Mod. Phys.* **29**, 269 (1957).
- [55] D. F. Kurdgelaidze and F. Funke, *Izv. VUZ, Fizika* **3**, 94 (1973); **4**, 7 (1973).
- [56] B. V. Danilyuk, *Izv. VUZ, Fizika* **9**, 75 (1974); **8**, 79 (1977).
- [57] H. P. Dürr, *Nuovo Cim.* **73A**, 165 (1983).
- [58] H. Saller, *Nuovo Cim.* **71A**, 17 (1982); **74A**, 159 (1983); **76A**, 663 (1983).
- [59] M. H. Nous, *Lett. Nuovo Cim.* **38**, 65 (1983).
- [60] M. A. Rodriguez and M. Lorente, *Nuovo Cim.* **83A**, 249 (1984).
- [61] N. A. Doughty and G. P. Collins, *J. Math. Phys.* **27**, 1639 (1986).
- [62] S. N. Gupta, *Phys. Rev.* **96**, 1683 (1954).
- [63] R. H. Kraichnan, *Phys. Rev.* **98**, 1118 (1955).
- [64] R. L. Arnowitt, *Phys. Rev.* **105**, 735 (1957).
- [65] S. Weinberg, *Phys. Rev.* **135**, B 1049 (1964); **138**, B 988 (1965).
- [66] W. Wyss, *Helv. Phys. Acta* **38**, 369 (1965).
- [67] V. I. Ogievetsky and I. V. Polubarinov, *Ann. Phys. New York* **35**, 167 (1965).
- [68] S. Deser, *Gen. Rel. Grav.* **1**, 9 (1970).
- [69] H. J. Groenewold, *Int. J. Theor. Phys.* **9**, 19 (1974).
- [70] D. G. Boulware and S. Deser, *Ann. Phys. New York* **89**, 193 (1975).
- [71] G. Papini and S. R. Valluri, *Phys. Rep.* **33**, 51 (1977).
- [72] J. J. von der Bij, H. van Dam, and Y. Y. Ng, *Physica* **116A**, 307 (1982).
- [73] P. Mittelstaedt, *Z. Physik* **211**, 271 (1968).
- [74] H. Stumpf, *Z. Naturforsch.* **38a**, 1064 (1983); **38a**, 1184 (1983); **39a**, 441 (1984).
- [75] H. Stumpf, *Z. Naturforsch.* **40a**, 14 (1985); **40a**, 183 (1985); **40a**, 294 (1985); **40a**, 752 (1985).
- [76] H. Stumpf, *Z. Naturforsch.* **41a**, 683 (1986); **41a**, 1399 (1986); **42a**, 213 (1987).
- [77] D. Grosser and T. Lauxmann, *J. Phys. G* **8**, 1505 (1982).
- [78] D. Grosser, B. Hailer, L. Hornung, T. Lauxmann, and H. Stumpf, *Z. Naturforsch.* **38a**, 1056 (1983).
- [79] B. Hailer, Thesis, University of Tübingen 1985; L. Hornung, Thesis, University of Tübingen 1986; T. Lauxmann, Thesis, University of Tübingen 1986; M. Rosa, Diplomarbeit, University of Tübingen 1987; D. Grosser, *Nuovo Cim.* **97A**, 439 (1987).
- [80] H. Stumpf, *Ann. Phys. (Germany)* **13**, 294 (1964); *Acta Phys. Austr. Suppl.* **9**, 195 (1972); *Z. Naturforsch.* **24a**, 188 (1969); **24a**, 1022 (1969); **25a**, 575 (1970); **30a**, 708 (1975); **36a**, 1024 (1981).
- [81] J. A. Wheeler, *Phys. Rev.* **52**, 1107 (1937); **52**, 1083 (1937).
- [82] K. Wildermuth and Y. C. Tang, *A unified theory of the nucleus*, Vieweg-Verlag, Braunschweig 1977.

- [83] E. Schmid, Phys. Rev. C **21**, 691 (1980).
- [84] P. Kramer, G. John, and D. Schenzle, Group theory and the interaction of composite nucleon systems, Vieweg-Verlag, Braunschweig 1981.
- [85] A. Faessler, Prog. Part Nucl. Phys. **11**, 171 (1984).
- [86] H. Müther, A. Polls, and T. T. S. Kuo, Nucl. Phys. A **435**, 548 (1985).
- [87] E. A. Lord, Proc. Cambr. Phil. Soc. **69**, 423 (1971).
- [88] W. Wilson, Gen. Rel. Grav. **12**, 51 (1980).
- [89] H. Dehnen and F. Ghaboussi, Nucl. Phys. B **262**, 144 (1985).
- [90] F. Gürsey, Nuovo Cim. **5**, 154 (1957).
- [91] G. Brauns, Z. Naturforsch. **19a**, 825 (1964); **20a**, 649 (1965).
- [92] B. K. Datta, Nuovo Cim. **6B**, 1 (1971); **6B**, 16 (1971).
- [93] F. W. Hehl and B. K. Datta, J. Math. Phys. **12**, 1334 (1971).
- [94] W. Ulmer, Int. J. Theor. Phys. **13**, 51 (1975).
- [95] M. Borneas, Int. J. Theor. Phys. **15**, 773 (1976); Analele Univ. Timisoara (Ser. fiz.) **20**, 85 (1982).
- [96] H. Stumpf, Z. Naturforsch. **37a**, 1295 (1982).
- [97] D. Grosser, Z. Naturforsch. **38a**, 1293 (1983).
- [98] H. M. Fried, Functional methods and models in quantum field theory, MIT-Press, 1972.
- [99] S. Weinberg, Gravitation and Cosmology: Principles and applications of the general theory of relativity, Wiley, New York 1972.
- [100] A. Lichnerowicz, C. R. Acad. Sci. Paris **222**, 432 (1946).
- [101] S. B. Edgar, Gen. Rel. Grav. **12**, 347 (1980).